

Few-Nucleon Systems in a Quirky World: Lattice Nuclei in Effective Field Theory

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Abstract

I describe how nuclear structure can be predicted from lattice QCD through low-energy effective field theories, using as an example a world simulation with relatively heavy up and down quarks.

1 Introduction

Weakly bound systems are fascinating for the surprising quantum features they display regardless of the details of their short-distance structure. They beg for a description with effective field theory (EFT), because that is the general framework to turn separated physics scales into a controlled expansion based on symmetries, rather than details of the dynamics. I have been smitten with EFT since shortly after leaving São Paulo for my Ph.D. in Austin. Only much later did I learn that a master of weakly bound systems (halo nuclei [1], neutron-rich nuclei [2], atoms near Feshbach resonances [3],...) lived in my hometown. This contribution is dedicated to this master, Professor Mahir Saleh Hussein, on the occasion of his 70th birthday.

I will argue here, based on the work of Refs. [4, 5, 6], that recent lattice QCD (LQCD) data [7, 8, 9, 10] suggest that light nuclei are weakly bound even in a world with relatively large quark masses. It seems that these data can be described with an EFT, Pionless or Contact EFT [11, 12, 13], where all degrees of freedom except nucleons are implicit [14]. This is, *mutatis mutandis*, the same theory that one can use to describe other systems and processes characterized by relatively large sizes: halo nuclei such as ${}^6\text{He}$ [15], shallow molecules such as He_4 trimers [16], atom recombination near Feshbach resonances [17], *etc.* It has been known for some time to apply to light nuclei at the physical value of the quark masses [18, 19, 20, 21, 22, 23]. If more or less familiar nuclear structure can be predicted for larger quark masses, perhaps nuclear physics is less accidental than we are used to think.

This is not just an academic exercise. As quark masses decrease in LQCD, pions become lighter and another, less universal but more predictive EFT, Pionful or Chiral EFT [14, 24], can be used to connect results at different quark masses [25]. We will at that point be able to predict even more deeply bound nuclei from LQCD, following the same steps as in Refs. [4, 5, 6], just with Pionless EFT replaced by Chiral EFT.

After describing the lattice world in Sec. 2 and reviewing Pionless EFT in Sec. 3, the main results for lattice nuclei are summarized in Sec. 4. An outlook is offered in Sec. 5. I hope Hussein is pleased with another unexpected gift offered by weakly bound systems.

2 EFT and QCD

It is intuitively clear, and supported by our experience in physics, that only certain degrees of freedom and symmetries are relevant at a given distance scale. Less obvious, but equally supported by the existence of virtual processes in quantum mechanics, is that all interactions allowed by symmetry take place among the relevant degrees of freedom. EFT is simply the framework that incorporates these facts. To be predictive, the basic assumption (“naturalness”) is that once a few scales are identified, the infinite number of interaction strengths, or “low-energy constants” (LECs), can be written as combinations of these scales, times numbers of $\mathcal{O}(1)$. As a consequence, observables, which can be expressed through T matrices for various processes, can be obtained as controlled expansions in Q/M , where Q represents the momenta of interest and comparable mass scales,

and M , high mass scales. (I use natural units where $\hbar = c = 1$.) Because all interactions are included, observables are renormalization-group (RG) invariant, that is, independent of the arbitrary regularization procedure used to separate explicit from implicit degrees of freedom. For a basic introduction to EFT, see Ref. [26].

Nuclear physics is well described in terms of nucleons subject to (possibly approximate) Lorentz, (possibly approximate) baryon-number, (approximate) time-reversal, and (approximate) parity invariance. QCD for the two lightest quark flavors, as relevant for nuclear physics, has essentially two separate scales:

1. $M_{QCD} \sim 1$ GeV, where the coupling constant of QCD formulated in terms of quarks and gluons becomes large. M_{QCD} sets the scale for hadronic masses, including the nucleon mass m_N , and for $4\pi f_\pi$, where f_π is the radius of the “chiral circle” formed by the set of $SU(2)_L \times SU(2)_R \sim SO(4)$ minima of the QCD effective potential. Picking one of these minima leads to spontaneous symmetry breaking, the emergence of pions as Goldstone bosons, and the manifestation of $f_\pi \simeq 90$ MeV as the pion decay constant.
2. $\bar{m} \sim 5$ MeV, the average quark mass, which explicitly breaks chiral symmetry, creates an absolute minimum of the QCD effective potential, and endows pions with a mass $m_\pi^2 = \mathcal{O}(M_{QCD}\bar{m})$. \bar{m} also affects most other quantities, including m_N . We can trade \bar{m} for m_π .

Contrary to real experiments, LQCD simulations can probe worlds where m_π takes different values. In fact, the high cost of light quarks roaming the lattice constrains present calculations to large values of m_π . While it certainly is a disadvantage that LQCD cannot reach realistic values yet, one can turn a disadvantage into an advantage by learning how nuclear physics depends on m_π .

Existing laboratory and LQCD data for nucleon masses and light-nuclear binding energies are summarized in Tab. 1. I also list the EFT results described in Secs. 3 and 4. The LQCD calculations are performed with equal light quark masses and no photons, hence isospin symmetry is exact and the binding energy $B_A(S, I)$ is determined by the nucleon number A and the spin-isospin combination S, I of the state. It did not have to be so, but masses and binding energies in Tab. 1 increase more or less monotonically with m_π . Note that Ref. [27] finds no bound states in a large range of pion masses that includes the values in Tab. 1. Since the model-independence of some of its results, obtained through a non-observable potential, remains a question mark, I do not consider these data here.

From the experimental and LQCD data in Tab. 1, we can infer the relevant momentum scales for real and lattice nuclei, which I list in Tab. 2. Besides pion and nucleon masses, the momentum associated with the excitation of the lowest baryon, the Delta isobar, in two-nucleon scattering [28] is also given, using the LQCD data for the Delta mass m_Δ compiled in Ref. [29]. As one can see, for the three values of the pion mass the typical nuclear momenta $\sqrt{2m_N B_A/A}$ for $A = 2, 3, 4$ are much smaller than m_N , suggesting that a description in terms of non-relativistic nucleons is always appropriate.

EFTs for non-relativistic nucleons have some simple, useful features. Pair creation is a short-range effect and the theory can be formulated in terms of Pauli spinors representing

Table 1: Neutron and proton masses, and binding energies of the lightest nuclei at various values of the pion mass. All entries are in MeV. The first column summarizes experimental data, the third [10], fourth [7] and fifth [8] columns give LQCD data, and the second [23] and sixth [4] columns show EFT input (marked with *) and results. The EFT calculations are discussed in Secs. 3 and 4.

m_π Nucleus	140 [nature]	140 [23]	300 [10]	510 [7]	805 [8]	805 [4]
n	939.6	939.0 *	1053	1320	1634	1634 *
p	938.3	939.0	1053	1320	1634	1634
^2n	—	—	$8.5 \pm 0.7^{+2.2}_{-0.4}$	7.4 ± 1.4	15.9 ± 3.8	$15.9 \pm 3.8 *$
^2H	2.224	2.224 *	$14.5 \pm 0.7^{+2.4}_{-0.7}$	11.5 ± 1.3	19.5 ± 4.8	$19.5 \pm 4.8 *$
^3n	—	—				< 12.1
^3H	8.482	8.482 *	$21.7 \pm 1.2^{+5.7}_{-1.6}$	20.3 ± 4.5	53.9 ± 10.7	$53.9 \pm 10.7 *$
^3He	7.718	8.482	$21.7 \pm 1.2^{+5.7}_{-1.6}$	20.3 ± 4.5	53.9 ± 10.7	53.9 ± 10.7
^4He	28.30	28.30 *	$47 \pm 7^{+11}_{-9}$	43.0 ± 14.4	107.0 ± 24.2	89 ± 36
$^4\text{He}^*$	8.09	10 ± 3				< 43.2
^5He	27.50					98 ± 39
^5Li	26.61					98 ± 39
^6Li	32.00	23 ± 7				122 ± 50

Table 2: Momentum scales for various quark masses: nucleon mass, effective momentum for Delta isobar excitation, pion mass, and effective binding momenta for S -shell nuclei. All entries are in MeV.

m_N	1000	1300	1600
$\sqrt{2m_N(m_\Delta - m_N)}$	750	900	800
m_π	140	500	800
$\sqrt{2m_N B_A/A}$ ($A=2-4$)	50-110	130-170	190-300

forward propagation in time. The EFT Lagrangian contains the usual non-relativistic kinetic terms in lowest order, with relativistic corrections implemented at higher orders in a Q/m_N expansion. Nucleon energies are of $\mathcal{O}(Q^2/m_N)$. Loops with antinucleons never need to be considered explicitly, and an N -body force does not affect the n -body system for $n < N$. At the N -nucleon level, all possible operators involving up to $2N$ nucleon fields are included, with an increasing number of derivatives. These interactions are highly singular and require regularization via some ultraviolet (UV) cutoff Λ . The explicit dependence on non-negative powers of Λ coming from loops is eliminated by renormalization of the LECs. Once (and only once) this is done at a given order, an integration over momenta in intermediate states contributes a factor $\mathcal{O}(Q^3/4\pi)$ to the T matrix. These factors of Q , m_N and 4π , together with the sizes of the renormalized LECs, are the ingredients to build the Q/M expansion.

In nature, there is a large separation of scales, $m_\pi \ll M_{QCD}$. A nucleon thus consists of an outer cloud of pions at distances $\sim 1/m_\pi$ surrounding an unresolved, dense core of size $\sim 1/M_{QCD}$. Empirically, nuclear sizes scale as $R_A \sim A^{1/3}r_0$, where $r_0 \sim 1.2$ fm is possibly set by a combination of f_π and m_π . Nuclei are expected to be large on the $1/M_{QCD}$ distance scale because of the two effects: the size of the pion cloud around each nucleon and the piling up of nucleons. LQCD has to fight both effects to contain nuclei within the lattice length $L \gtrsim R_A$, while striving for a much smaller short-distance regulator in the form of a lattice spacing $b \lesssim 1/M_{QCD}$. EFT offers a strategy to extrapolate QCD to the large distances involved in nuclear physics: *i)* calculate with LQCD A -nucleon observables for $A = 2, 3, 4$; *ii)* calculate the same observables with EFT and match LQCD, thus determining the LECs; and *iii)* solve the EFT for $A \geq 5$ using the powerful “*ab initio*” methods that have been developed in recent years, such as the no-core shell model (NCSM) [30], the effective-interaction hyperspherical harmonics (EIHH) [31], and the auxiliary-field diffusion Monte Carlo (AFDMC) [32] methods.

For momenta $Q \sim m_\pi \ll M_{QCD}$, one can formulate an EFT, Chiral EFT [14, 24], which includes, in addition to nucleons, also pions and the lowest nucleon excitations. In this EFT, $M \sim M_{QCD}$, and one treats the inner nucleon cloud in a multipole-type expansion. The approximate chiral symmetry of QCD plays a crucial role, because it ensures that pions couple weakly at low momenta, which gives rise to a loop, or equivalently density, expansion for the larger, more sparse pion cloud. Chiral EFT allows one to, in principle, calculate the dependence of low-energy nuclear observables on m_π [25]. Unfortunately, however, an RG-invariant formulation of Chiral EFT is still work in progress [33]. This is not too serious a problem in the sense that it is doubtful that the Chiral EFT expansion holds at pion masses explored so far by LQCD. For example, studies of the convergence of Chiral EFT for $A = 0$ suggest a breakdown of the expansion at a pion mass no larger than 500 MeV [34]. When smaller pion masses can be reached and a proper formulation well developed, Chiral EFT could be used as a tool for extrapolation of nuclear quantities in m_π , as it is already for meson and one-nucleon observables. Chiral EFT LECs would then be determined from LQCD instead of experimental data, and a solution of Chiral EFT used to extrapolate LQCD to larger A .

3 Pionless EFT

In fact, Tab. 2 suggests that the extrapolation to larger A can already be performed. For all available pion masses, the typical nuclear momentum is not only much smaller than m_N , it is also smaller than m_π . This is already true in nature and, as the pion mass increases, pion effects become more short-ranged relative to nuclear distances. Assuming that the Delta continues to be the lowest baryon excitation, any effects from other hadrons are also short-ranged.

Thus, at momenta $Q \ll m_\pi$, nucleons should suffice as explicit degrees of freedom. In the appropriate EFT, Pionless EFT [14], $M \sim m_\pi$ and *all* interactions are of contact type, which in coordinate space give (renormalized versions of) delta functions and derivatives.

In nature, Pionless EFT describes well the properties of low-energy scattering and bound states for $A = 2$ [18], 3 [19, 20], 4 [21, 22], and even (but less well) 6 [23]. One expects Pionless EFT to breakdown at some point as nuclei get denser, but its reach is presently unknown.

In the two-nucleon sector, there are two independent non-derivative contact interactions, with renormalized LECs $C_{01} = \mathcal{O}(4\pi a_2(0, 1)/m_N)$ and $C_{10} = \mathcal{O}(4\pi a_2(1, 0)/m_N)$, in terms of the 1S_0 and 3S_1 scattering lengths, respectively $a_2(0, 1)$ and $a_2(1, 0)$. Together with the general estimates for nucleon energies and loop integrals given above, each iteration of this potential in the T matrix yields a factor $\mathcal{O}(Q|a_2|)$. When $|a_2| \gg 1/M$ as a consequence of a shallow pole in the T matrix, one needs to include all iterations for $Q \gtrsim 1/|a_2|$, namely solve the corresponding Schrödinger equation exactly. It is easy to show that the dangerous UV regulator dependence can be eliminated if the bare LECs $C_{SI}(\Lambda) \propto 1/\Lambda$. In coordinate space one can understand this by noticing that a delta function is a $\Lambda^3/4\pi$ singularity and overwhelms the kinetic term, which grows at most as Λ^2/m_N , unless the associated LEC goes as $4\pi/(m_N\Lambda)$. The relative $(a_2\Lambda)^{-1}$ corrections in $C_{SI}(\Lambda)$ provide then just the balance necessary for a low-energy (real or virtual) bound state with binding momentum $\mathcal{O}(1/a_2)$. The regularization procedure leaves behind a relative error of $\mathcal{O}(Q/\Lambda)$, which can be made arbitrarily small by taking Λ arbitrarily large. In leading order (LO) we obtain the first term in the effective-range expansion (ERE) of the T matrix. The residual cutoff dependence can be removed by the two-derivative interactions present in the same channels, which, as a consequence, have a natural relative size $\mathcal{O}(Q/M)$ with respect to LO. That means they make up the next-to-leading order (NLO), their renormalized LECs being of $\mathcal{O}(4\pi a_2^2 r_2/m_N)$, with effective ranges $|r_2| = \mathcal{O}(1/M)$. The argument can be generalized to more-derivative terms in the S waves, which contribute at progressively higher orders. In higher waves, where there seem to be no shallow poles, all LECs scale with M according to their canonical dimension, which means that no other wave needs to be considered up to N²LO. RG invariance requires that, as allowed by their small relative sizes, subleading orders be treated in perturbation theory. The gory details are spelled out in Ref. [11].

In this way, Pionless EFT generates an expansion of the two-nucleon amplitude equivalent [11] to the ERE. At LO, only the two non-derivative contact interactions need to be included. Each LEC is determined by one datum, say the scattering length or the binding momentum of the shallow pole (which differ only by higher-order terms). At NLO the two S -wave two-derivative contact interactions need to be included in first distorted-wave Born approximation, and each new LEC requires another datum, say the effective range. A nice description of two-nucleon scattering at physical quark mass was found in Ref. [18]. In the regime $1/|a_2| \ll Q \ll M$, the amplitude is approximately scale invariant and $SU(4)$ spin-isospin symmetric [35].

Nuclear-physics folklore would suggest that few-nucleon forces are of higher order. Indeed, in certain channels such as $^4S_{3/2}$ neutron-deuteron scattering, high accuracy can be obtained in the first orders [20]. However, in the $^2S_{1/2}$ channel [19] (and for three bosons in relative S waves [12]), the only way to eliminate non-negative powers of Λ (in particular, the “Thomas collapse” [36] of the ground state) is to ensure that the non-derivative six-

nucleon operator (there is only one) is present in LO. Again the cutoff dependence of its bare LEC, $D(\Lambda) \propto 1/\Lambda^4$, can be understood by a simple coordinate-space argument. When going from two to three bodies, the delicate balance between kinetic terms and two-body contact is destroyed because the number of kinetic terms doubles while the number of pair-wise interactions triples. The result is the Thomas collapse where the two-body attraction wins and leads to a three-body binding energy that grows as Λ^2/m_N . This growth has to be canceled by the contact three-body interaction; since it involves two delta functions, or $\Lambda^6/(4\pi)^2$, the bare LEC should be roughly $(4\pi)^2/(m_N\Lambda^4)$. The cutoff dependence of D is still more complicated, though. At very low cutoffs the three-body force might be attractive or repulsive, but once Λ^2/m_N exceeds the binding energy of the ground state, the three-body force must be increasingly repulsive to prevent the collapse. With a regulator procedure that preserves the approximate scale invariance of the two-body subsystems, at a critical Λ one can maintain the binding energy fixed only at the cost of making the three-body force attractive and accreting a very deep bound state. As the cutoff increases past critical, the two-body attraction continues to increase and the three-body force gets less attractive, till it becomes repulsive and a new cycle begins. In fact [12, 19] the bare LEC is on an RG limit cycle, approximate scale invariance is reduced to an approximate *discrete* scale invariance, and there is a tower of approximately geometric three-body “Efimov states” [37]. Varying the renormalized $D = \mathcal{O}((4\pi)^2 a_2^4/m_N)$ shifts the position of the tower and changes the three-body scattering length, leading to a correlation known as the Phillips line [38]. In nuclear physics the explicit breaking of scale invariance is such that the “tower” consists of a single state, the triton. The two-derivative three-body force first appears at N²LO [12, 19].

Calculations [13, 21], which are however somewhat limited in cutoff variation, indicate that four-body observables do not display non-negative powers of Λ up to NLO, in the absence of four-body forces. This is perhaps not surprising since the three-body force is effectively repulsive and the number of triplets grows faster than doublets. With fixed two-nucleon input, variation in D leads to a correlation between four- and three-body binding energies, the “Tjon line” [39], which passes close to the experimental point. Pionless EFT thus successfully postdicts the alpha-particle binding energy [21], and scattering can be calculated as well [22]. Notice, however, that the arguments above leave open the possibility of cutoff dependence in the regions where the three-body force is attractive, and indeed Ref. [40] found sensitivity in four-body properties to a four-body scale. As far as I can see, there is yet no compelling argument that this sensitivity is due to an *LO (or even NLO)* force. I will assume that the dominant four-body force, presumably coming from the single non-derivative eight-nucleon operator, first contributes beyond NLO. Since, thanks to the Pauli principle, five- or more-body forces involve at least two derivatives, they are likely of even higher order.

It is probably safe to assume that, up to NLO, Pionless EFT is renormalizable with, besides two-body forces, a single non-derivative three-body force. Its LEC is determined by one three-body datum, say the neutron-deuteron $^2S_{1/2}$ scattering length or the triton binding energy. Thus at LO (NLO) three (five) data are needed as input, in addition to the nucleon mass, and everything else is a prediction. Pionless EFT is not *just* the ERE;

it is the extension to few-body systems that preserves model independence. Pionless EFT accounts for a series of apparently unrelated, qualitatively unique phenomena, such as the Thomas collapse, Efimov states, the Phillips and Tjon lines, and presumably similar correlations for bigger systems. With a few adaptations, it applies to other systems characterized by a small ratio r_2/a_2 —see, *e.g.* Refs. [15, 16, 17]. The approximate discrete scale invariance has striking consequences for the spectrum of few-boson systems [41], where a state in Efimov’s three-body tower generates pairs of “image” states in bigger systems. The alpha-particle ground and excited states can likely be interpreted this way. Moreover, since the LO three-body force is $SU(4)$ symmetric, Pionless EFT provides a justification [12, 19] for the approximate $SU(4)$ symmetry proposed by Wigner [42].

It is unfortunately still unclear how far up the nuclear chart this EFT can be pushed. The only calculation [23] beyond the four-nucleon system was based on the NCSM [30], when the EFT Hamiltonian was diagonalized in a harmonic-oscillator basis. This basis has a natural UV cutoff in the form of a maximum allowed number of shells. It contains also an infrared (IR) cutoff provided by the spacing between shells. In the simplest approach, the LECs are fitted to the experimental binding energies of the lightest nuclei for every cutoff pair, and binding energies for larger nuclei are calculated and extrapolated to large UV and small IR cutoffs. Results of an LO calculation [23], where the deuteron, triton, and alpha-particle ground-state energies were used as input in addition to the nucleon mass, are shown in Tab. 1. The input data are indicated by a “*” in Tab. 1. Estimating the error as 30% from r_2/a_2 in the 3S_1 channel, one sees that the excited state of the alpha particle is postdicted very well, while ^6Li is barely consistent. However, the error could be as large as 80% if we consider the ratio in Tab. 2 between alpha-particle momentum and pion mass. Higher-order calculations are clearly needed.

4 EFT for Lattice Nuclei

As noted in Ref. [4], the widening gap shown in Tab. 2 between pion mass and typical nuclear momentum implies Pionless EFT should work better at larger pion masses. In the first calculation ever to fit lattice *nuclear* data, Ref. [4] used the nucleon mass and light-nuclear binding energies at the highest pion-mass value, $m_\pi = 805$ MeV, from the NPLQCD collaboration [8] as input for Pionless EFT in LO. A calculation using the $m_\pi = 510$ MeV binding energies from Ref. [7] is in progress [5]. The existence of a dineutron bound state allows the use of its binding energy instead of the alpha-particle’s as input.

From Tab. 2 a conservative estimate for the Q/m_π expansion parameter at $m_\pi = 805$ MeV is 40%. Two-nucleon scattering lengths and effective ranges are also available at this pion mass [9], and are consistent with an almost degenerate double bound-state pole in the T matrix of each S wave, which is thought to be incompatible with a short-range non-relativistic potential [43]. References [4, 5] assume $|r_2| = \mathcal{O}(1/m_\pi)$, as for physical quark masses. If one uses the ratio r_2/a_2 , the error estimate for LO is instead 50%. A test of convergence will have to await an NLO calculation.

In a renormalizable theory, only convenience guides the choice of regulator. In the present case, we want, as in Ref. [44], a local potential that allows the use of many-body techniques that cannot handle non-local interactions well, such as AFDMC [32]. This can be achieved with a regulator function $f(\vec{q}^2/\Lambda^2)$ in the momentum transfer \vec{q} , or its Fourier transform $F(r^2\Lambda^2)$ in terms of the radial coordinate r . Ref. [4] employed two forms, $f_n(x) = \exp(-x^{2n})$ with $n = 1, 2$, which get increasingly closer to a sharp regulator. The isospin-symmetric Hamiltonian can be written in coordinate space as

$$\begin{aligned}
H = & -\frac{1}{2m_N} \sum_i \nabla_i^2 + \frac{1}{4} \sum_{i < j} [3C_{10}(\Lambda) + C_{01}(\Lambda) + (C_{10}(\Lambda) - C_{01}(\Lambda)) \vec{\sigma}_i \cdot \vec{\sigma}_j] F(r_{ij}^2 \Lambda^2) \\
& + \sum_{i < j < k} \sum_{cyc} D(\Lambda) \vec{\tau}_i \cdot \vec{\tau}_j F(r_{ik}^2 \Lambda^2) F(r_{jk}^2 \Lambda^2) + \dots,
\end{aligned} \tag{1}$$

where $\vec{\sigma}_i/2$ ($\vec{\tau}_i/2$) is the spin (isospin) of nucleon i , \sum_{cyc} stands for the cyclic permutation of a particle triplet (ijk) , and “...” for terms containing more derivatives and/or more-body forces. The LECs m_N , $C_{10}(\Lambda)$, $C_{01}(\Lambda)$, $D(\Lambda)$, *etc.* depend on m_π , since pions are part of the short-distance physics not included explicitly. The m_π dependence of two- and three-nucleon observables in Pionless EFT has been studied with input from Chiral EFT in Ref. [45].

The two-nucleon Schrödinger equation was solved [4] for the LO Hamiltonian with the Numerov method, and $C_{10}(\Lambda)$ and $C_{01}(\Lambda)$ fitted to the deuteron $B_2(1, 0)$ and dineutron $B_2(0, 1)$ binding energies [8], respectively. The cutoff dependence of the LECs is found to be qualitatively similar to other regulators [11, 18]: $C_{SI}(\Lambda)\Lambda$ approaches a regulator-specific constant at a rate determined by $\sqrt{m_N B_2(S, I)}$. For large cutoffs one should have in LO $a_2(1, 0) \approx 1/\sqrt{m_N B_2(1, 0)} \simeq 1.12$ fm. For cutoff variation in the range 2-14 fm⁻¹, Ref. [4] finds explicitly $a_2(1, 0) = (1.2 \pm 0.5)$ fm ((1.1 ± 0.1) fm) with the regulator f_1 (f_2). For comparison, LQCD gives $a_2(1, 0) = 1.82^{+0.14+0.17}_{-0.13-0.12}$ fm directly [9]. The situation is similar in the 1S_0 channel.

For systems with $3 \leq A \leq 6$ nucleons the Schrödinger equation was solved [4] with the EHH method, where the wavefunction is expanded into a set of antisymmetrized hyperspherical-harmonic spin-isospin states. Convergence is controlled by the hyper-angular quantum number K_{max} , results being obtained by extrapolation to the limit $K_{max} \rightarrow \infty$ [31]. The corresponding error was estimated to be smaller (for the lighter systems, much smaller) than the EFT truncation error. For systems with $A \geq 4$ the AFDMC method was also used. In this technique [32], the ground-state energies are projected from an arbitrary initial state by means of a stochastic imaginary-time propagation. The numerical simulations are simplified by the introduction of auxiliary fields via a Hubbard-Stratonovich transformation. In all these calculations [4], the regulator employed was f_1 with $2 \leq \Lambda$ fm ≤ 8 .

The LEC $D(\Lambda)$ was determined [4] imposing that the $^3\text{H}/^3\text{He}$ binding energy B_3 is reproduced at any value of Λ . It was found that $D(\Lambda)\Lambda^4$ approaches a finite limit, as for other regulators [12, 19]. The limit-cycle behavior is not seen, as in other cases when the number of three-body bound states is kept fixed with regulators that do not preserve the

approximate scale invariance of the two-body subsystems, *e.g.* Ref. [46]. With LECs thus fixed, a complete LO potential is available to predict other properties of lattice nuclei.

The four-nucleon system, solved [4] with both EIHH and AFDMC methods, provides a consistency check between the two *ab initio* methods, and between them and LQCD. The two *ab initio* methods produced results for the ${}^4\text{He}$ binding energy B_4 that agree well within the (large) LQCD error. In either case, B_4 was found to depend only weakly on the cutoff, changing by about 20% when Λ grows by a factor of 4. Over a wide cutoff range the EFT prediction reproduces the LQCD result within its error, evidence that the EFT in LO captures the essence of the strong-interaction dynamics. As D varies (at fixed Λ) within the error bars of B_3 , B_4 also changes within its error bars. The estimate of a 40% error in LO EFT is likely conservative, indeed.

The power of EFT is the relative ease with which it can be solved for more-body systems. Binding energies for $A \geq 5$ are predictions that extend LQCD into new territory. Using $\Lambda = 2\text{ fm}^{-1}$, the authors of Ref. [4] searched unsuccessfully for excited states in $A = 2, 3, 4$ systems. Similarly, they found no evidence of ${}^3\text{n}$ droplets, for which the ground-state binding energy coincided with the two-body threshold. Results [4] for the $A = 5, 6$ ground states at $\Lambda = 2\text{ fm}^{-1}$ are shown in Tab. 1, with errors estimated from the EFT truncation. For ${}^5\text{He}$ a bound state with binding energy $B_5 = 98.2\text{ MeV}$ for $\Lambda = 2\text{ fm}^{-1}$ coincided with the four-body threshold for $\Lambda = 4\text{ fm}^{-1}$. The ${}^6\text{Li}$ ground state for $\Lambda = 2\text{ fm}^{-1}$ was found at $B_6 \approx 122\text{ MeV}$. In this case the error in K_{max} extrapolation was about 3 MeV, which is somewhat larger than for lighter systems but still small compared with input and truncation errors. Calculations with AFDMC at larger A are in progress [6].

These results are afflicted by considerable error bars. Even though the 40% assigned to the EFT expansion is likely an overestimate, the LQCD input itself has large uncertainties of about 25%. With this caveat, the trend of the results is surprising. There is a qualitative difference with $A = 2$ at the physical pion mass, because the dineutron is bound at larger masses. (For the effects of the dineutron scattering length on light nuclei in Pionless EFT at physical pion mass, see Ref. [47].) But this is consistent with other binding energies, which are all larger, and all larger by roughly similar amounts. The gap at $A = 5$, familiar in nature, seems to survive the increase in pion mass. And $B_6/6 \approx 20\text{ MeV}$, similar to lattice ${}^4\text{He}$ for which $B_4/4 \approx 25\text{ MeV}$. This suggests that nuclear saturation might not be tremendously sensitive to the pion mass. Overall, it seems that the lattice world at $m_\pi = 805\text{ MeV}$ is not that different from our own, with B_A/A scaled by a factor 4 or 5.

5 Outlook

Natural light ($A \leq 4$) nuclei are halo-type systems in the sense that they have sizes large compared the range of the force, and are thus described by Pionless EFT. One might expect this feature to result from fine-tuning, and to find very different worlds at unphysical pion masses. Surprisingly, the first LQCD calculations and their EFT extrapolations seem to suggest the opposite. Perhaps pions do not play as decisive a role

in low-energy QCD as we are used to think, and some of the defining properties of nuclei are relatively insensitive to the value of the pion mass. If this is true, it should have implications for the use of nuclei in tests of the variability of fundamental constants [48].

Of course, at this point these are only hints. The same exercise can be, and is being [5], carried out in LO with the LQCD data at $m_\pi = 500$ MeV [7]. EFT extrapolations can be repeated at other pion masses as LQCD results appear. More urgently, EFT calculations need to be performed for $A \geq 4$ at higher cutoffs to confirm renormalizability, even at the physical pion mass. The scattering lengths and effective ranges at $m_\pi = 800$ MeV [9] give just enough input for an NLO analysis, which should allow stronger statements about the convergence of Pionless EFT at unphysical pion masses. Finally, at all values of pion mass, one should increase A [6] to confirm trends in B_A/A and to seek the limit of applicability of the EFT.

On a longer time frame, as pion masses in LQCD drop sufficiently, one can use Chiral EFT to extrapolate further down in pion mass and fully predict real nuclei, following the same steps as in Refs. [4, 5, 6]. It is a lot to do, but it promises to fulfill a longstanding dream of nuclear effective field theorists, teaching us much about the connection between QCD and nuclear physics.

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